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On a Recent Attempt to Define the Interpretation Basis in the Many Worlds Interpretation of Quantum Mechanics

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David Deutsch's 1985 algorithm to determine the preferred product structure and basis in the many worlds interpretation is examined. His heuristic argument for the conditions appearing in the algorithm is found wanting. The question of the existence and uniqueness of a final solution to his algorithm is discussed. The algorithm is shown to be inadequate to account for certain types of measurement which are, at least in principle, possible.

1. INTRODUCTION: THE MANY WORLDS INTERPRETATION

The original idea behind the many worlds interpretation was suggested by Everett in his relative state formulation of quantum mechanics (Everett, 1957). DeWitt then elaborated the idea and introduced the many worlds terminology to the public (DeWitt, 1968). Since then several versions of the many worlds interpretation have been expounded. In this paper we are particularly concerned with that presented by Deutsch (Deutsch, 1985).

Proponents of the many worlds interpretation (MWI) are concerned to find a realist interpretation of quantum mechanics which, while consistent with our experience, takes quantum mechanics to be a complete, universal, physical theory and which does not rely on any *a priori* metaphysics to interpret any state function. The interpretation should not have to appeal to the existence of some system necessarily outside the quantum mechanical description, such as an observer, a measuring apparatus, or a macroscopic system, in order to interpret the state function of a system. An interpretation of quantum mechanics as a complete universal physical theory is desirable, for only such an interpretation can be used, without introducing hidden variables, to interpret the state function of the universe which appears in quantum cosmology and quantum gravity, and to make sense of applying

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quantum theory without a projection postulate to an isolated system consisting of an object and an apparatus successfully making measurements on the object.

If no projection postulate is assumed in the quantum mechanical description of measurement for an isolated object–apparatus system, then, if the initial state of the object system is not an eigenstate of the object observable being measured, the final state of the joint object–apparatus system is not a product of an object system state and an apparatus system state describing the apparatus as having some definite measurement outcome, nor is it a proper mixture of such states. Rather, it is a correlated "entangled state" for the two subsystems (or mixture of such entangled states): that is, a state in which neither the object system nor the apparatus system is in a definite state. Furthermore, the final state is not an eigenstate of the apparatus "pointer observable" defined for the joint system. This has been shown, assuming unitary evolution for the joint object–apparatus system, even under very general conditions for measurement (Fine, 1970; Brown, 1986). Such a final description seems to be inconsistent with our experience of definite measurement outcomes. Many attempts have been made to resolve this conflict between the quantum mechanical description and our experience. Following von Neumann (1955, Chapter V, Section I), the standard solution is to add an extra postulate to the theory of quantum mechanics, to the effect that, at some stage (not specified unambiguously by anyone as yet) in the measurement or observation the state function "collapses" to only one of the elements of the superposition.

Others have proposed that quantum mechanics does not provide a complete description of the world; the theory needs to be supplemented by further variables—"hidden variables"—which, with the quantum mechanical state, determine the final position of the apparatus pointer. This would allow in principle that the apparatus pointer may have a definite position even when its quantum mechanical state is not an eigenstate of the apparatus pointer observable or a proper mixture of such states. However, such theories are not without their own problems [see, for example, Bell (1964), Bub (1976), and Heywood and Redhead (1983); for a good survey of hidden variable theories see Belinfante (1973)]. The many worlds interpretation attempts to resolve the conflict without introducing either a new type of dynamics to collapse the problematic superposition or a new set of state variables. Instead, it hopes to provide a new interpretation of the superposition which is consistent with our experience of definite measurement outcomes.

According to DeWitt's version of the MWI, during the course of a measurement, the world splits into a number of different worlds, one for each element of the superposition obtained when the final state for the object–apparatus system, say $|\psi\rangle$, is expanded in the basis of product states of eigenstates of the apparatus pointer observable, say $\{|A_i\rangle\}$, and the normalized relative states for the object system, $\{N_i\langle A_i|\psi\rangle\}$. In each world, the apparatus pointer has a definite position indicating the eigenvalue of the apparatus pointer observable for the element of the superposition describing that world, and the object system is characterized by its normalized relative state for that world. In a "measurement of the first kind," these object system states will be eigenstates of the object observable being measured. Therefore, according to the MWI, at the end of such a measurement this observable will have a well-defined value in each of the worlds even if it did not have well-defined values at the start of the measurement.

As far as the quantum mechanical description is concerned, a measurement interaction is a type of quantum mechanical interaction between two systems and no particular formal characteristic picks out an actual measurement interaction from a measurementlike interaction. Therefore, DeWitt postulates that a splitting of worlds occurs as a result of any measurementlike interaction taking place anywhere in the universe. He suggests that the worlds produced in the splitting are such that in each world the mean square deviation of any "macroscopic" observable is never large (DeWitt, 1971).

One of the fundamental problems with the pre-Deutsch versions of the many worlds interpretation is that in order to determine how the universe is split for a given state $|\psi\rangle$, information beyond that given in the quantum mechanical description is required. The interpretations do not escape the use of *a priori* metaphysics as had been hoped. For example, in DeWitt's version, in order to interpret the state function at the end of a measurement, we need to know which subsystems in the Hilbert space for the system represent the object and apparatus subsystems, and which operator for the apparatus subsystem represents the position of the apparatus pointer. For a general state $|\psi\rangle$, not necessarily the result of a measurement, DeWitt's treatment suggests that we need to know which subsystems in the Hilbert space represent macroscopic systems and which operators represent macroscopic observables, in order to be able to determine which expansion of $|\psi\rangle$ should be taken to show how the worlds are split for that state. However, no characterization of the quantum mechanical description of these subsystems and observables is given. Therefore, the way the world is split for a given state $|\psi\rangle$ cannot be determined from the quantum mechanical description alone; we have to decide which subsystems and operators represent macroscopic systems and observables. Therefore, we rely on *a priori* metaphysics for the interpretation of $|\psi\rangle$, and it is left a mystery as to how the world in state $|\psi\rangle$ knows how to split according to our *a priori* metaphysics.

If collapse theories are not to rely on *a priori* metaphysics for their application, they, too, must give some means of specifying when and into which mixture the state collapses. But in their case this specification need only be within the vocabulary and dynamics of *some* physical theory, not necessarily quantum mechanics. The theory may be a new one, introduced to describe the process of the collapse. In a hidden variable theory, the problem of specification is overcome if the variables, together with the state function, determine which observables of the system have well-defined values at any time.

In his paper Deutsch (1985) attempts to provide a specification of the preferred subsystem structure and basis for the MWI within the theory of quantum mechanics. He sets up an algorithm which requires as input only the state function, Hilbert space, and Hamiltonian for the system at time t. As output it is supposed to give a particular product structure, that is, a division of the system into particular subsystems each associated with a particular subspace of the Hilbert space and a particular set of operators (the set of operators confined to that subspace), and a particular basis for each subsystem in that product structure. The basis so determined in the Hilbert space of the whole system Deutsch calls the "interpretation basis." It is this basis which is to be used in interpreting $|\psi\rangle$. It determines which observables have values in the many universes making up the world at time t. With the state function for the system, it determines the proportion of universes in which the different values for these observables occur. Before we begin a discussion of Deutsch's algorithm to determine the interpretation basis and his heuristic argument for the conditions appearing in the algorithm, let us draw attention to a further important difference between Deutsch's version of the MWI and the versions which appeared previously to his.

Everett (1957), Graham (1970), and DeWitt (1968) each thought that they had derived the statistical assertions of quantum mechanics in the context of the MWI without introducing any statistical postulates or probability assumptions. However, as has been pointed out by several writers following Ballentine (1973; see, for example, Bell, 1981; Deutsch, 1985; Tipler, 1986), a probability assumption has to be made in each of their derivations; each of them has to assume that worlds of measure nearly zero or zero in the norm Hilbert space sense have probability nearly zero or zero, respectively. These assumptions are covered by the assumption that the probability of a world described by an element of the superposition with measure $|c_i|^2$ is $|c_i|^2$. But even with this assumption, a problem still occurs in their derivations of the statistical assertions of quantum mechanics—how to understand statements about the probability of DeWitt worlds. It is assumed that each element in the appropriate superposition represents a separate DeWitt world (A1), and that the weights for each element represent the probability of that world (A2). But if each of these worlds definitely occurs and each only singularly (A1), then, if we are going to introduce probabilities, each world should be assigned the same probability value. (Whether this should be 1 or $1/n$, where n is the number of distinct worlds, is then open to debate.) There is nothing in the physical picture to correspond to the claim (A2) that the different worlds have certain probabilities, the values of which vary according to the measure in Hilbert space associated with each world.

Deutsch thought that he could overcome this problem by enriching DeWitt's picture of the many worlds: Deutsch postulates that the world consists of a continuously infinite measured set of universes (a set of universes together with a measure on that set). Given that the interpretation basis at time t is $\{|\alpha_i, t\rangle\}$, he postulates that the world described by the state function $|\psi\rangle$ at time t is such that, in a proportion $|\langle\psi|\alpha_i, t\rangle|^2$ of all the universes, the value of any observable $\hat{O}$ diagonal in the interpretation basis is $\langle\alpha_i, t|\hat{O}|\alpha_i, t\rangle$. Therefore, if the expansion of the world state function in the interpretation basis $\{|\alpha_i\rangle\}$, $i=1, \ldots, n$, is $|\psi\rangle = \sum_i c_i |\alpha_i\rangle$, then the set of universes consists of n disjoint subsets. Each of these subsets Deutsch calls a "branch." The ith branch with measure $|c_i|^2$ contains a proportion $|c_i|^2$ of the universes. In other words, the measure associated with each branch is taken to represent the proportion of universes in that branch. Therefore, in Deutsch's version of the many worlds interpretation (perhaps better called a many universes interpretation), there is at least something in the physical picture to which to relate the probabilities associated with the Hilbert space measure, namely the proportion of universes in a branch. Whether this relationship between probability and proportion can be used to explain successfully, in a manner consistent with our ideas about probability, the probability statements we use to describe the phenomena that we experience in one universe in the set is a topic for discussion which we save for a further paper. Now we return to the main concern of this paper—a discussion of Deutsch's algorithm for the interpretation basis, and his heuristic argument for the algorithm.

2. DEUTSCH'S ALGORITHM TO DEFINE THE INTERPRETATION BASIS

Deutsch's argument for the conditions appearing in his algorithm to determine the interpretation basis is heuristic. It could not be otherwise unless we were to change the theory of quantum mechanics by adding further axioms. Using the Hilbert space, state function, and Hamiltonian for a system, we can set up definitions for many different bases in the Hilbert space. In order to decide which definition picks out the basis appropriate for the interpretation basis, considerations outside the theory of quantum mechanics must be invoked. This Deutsch does in his heuristic argument; he attempts to give physical reasons for why the conditions appearing in his algorithm should be satisfied by the interpretation basis. In Section 2.1 we criticise Deutsch's heuristic argument. These criticisms of course cannot show that his algorithm is definitely wrong, only that it is not set up by the argument he gives. The real test of the algorithm is whether, when it is conjoined with quantum theory and Deutsch's many universes interpretation, the resulting interpreted theory can account for the experiences we actually have. In Section 2.2, we consider the question of the existence and uniqueness of the solution to Deutsch's algorithm, and in Section 2.3 we show that his algorithm is inadequate to account for certain types of measurement, which are, at least in principle, possible.

2.1. Deutsch's Heuristic Argument for the Algorithm

First Deutsch considers the case of a measurement of the first kind. For such a measurement, we know what we want the preferred product structure and basis to be at the end of the measurement; we want the product structure which has the object as one subsystem, the apparatus as the other subsystem, and the basis of product states of the eigenstates of the object observable being measured and the eigenstates of the apparatus pointer observable. We require that certain features hold for measurements of the first kind. Deutsch claims that from these it follows that a certain condition is satisfied by the product structure and basis we want to specify as the interpretation basis in this case: there exist some $\hat{H}_1$ and $\hat{H}_2$, Hermitian operators confined to the object subsystem and apparatus subsystem, respectively, such that $\hat{H}-\hat{H}_1-\hat{H}_2$ is diagonal in the interpretation basis, where $\hat{H}$ is the Hamiltonian for the system. He then assumes that in general this condition holds for the interpretation basis: for a system consisting of two subsystems, the interpretation product structure and basis are such that there exist Hermitian operators $\hat{H}_1$ and $\hat{H}_2$ confined to the product structure subsystems, for which $\hat{H}-\hat{H}_1-\hat{H}_2$ is diagonal in the interpretation basis, where $\hat{H}$ is the Hamiltonian for the joint system (Condition C1). This condition is not sufficient to determine the interpretation basis on its own, but Deutsch claims that essentially it fixes the product structure, given a system which consists of two subsystems, the dimensionalities of whose Hilbert spaces is known.

For the second stage of his heuristic argument, Deutsch assumes that the product structure dividing a system into two subsystems is already given, and uses an argument based on external measurements to try to demonstrate that a second condition must be satisfied by the interpretation basis in this case. He argues that this condition must be satisfied if the interpretation using this basis is to remain consistent with the requirement that no effect can be propagated at superluminal velocities, or in general by means not described in the dynamics of quantum theory. He then assumes that this condition holds in general for the interpretation basis: for a system consisting of two subsystems, the interpretation basis is the basis made up of the product states of the eigenstates of the two subsystem density operators (Condition C2). Conditions C1 and C2 are to be used to determine the interpretation basis at any time from the state function, Hilbert space, and Hamiltonian for the system at that time, given that the system consists of just two subsystems, the dimensionality of whose Hilbert spaces is known.

Next Deutsch considers how these two conditions should be used to specify the interpretation basis for a system composed of three subsystems, the dimensionality of whose Hilbert spaces is known. He then suggests that the method admits of only one generalization to the case where the dimensionality of the Hilbert space for the whole system only is given and it has arbitrarily (but finitely) many factors. Deutsch assumes that the subsystems for such a system will have Hilbert spaces of prime dimension and that his algorithm will determine these subsystems through repeated use of Condition C1. In the case of a system with Hilbert space of dimension n, where n is the product of more than two primes, the algorithm instructs us first to take an arbitrary product structure dividing the system into two subsystems, one of which has a Hilbert space of prime dimension, and then to take further successive product structures dividing the "nonprime subsystem" into two subsystems, one with a Hilbert space of prime dimension, etc., until both subsystems have Hilbert spaces of prime dimension. Condition C2 is used to determine the basis for each of the "prime subsystems." The product structure itself is then determined by repeated use of Condition C1. In this way, the algorithm is supposed to determine the interpretation basis for *any* system, at *any* time, solely from its Hilbert space, state function, and Hamiltonian for that time. Although the generalization of Deutsch's method to the case of systems composed of more than two subsystems requires further explanation and discussion than is given in his article, we shall concentrate on the first two stages of Deutsch's heuristic argument, for it is in these stages that crucial gaps in his argument occur.

The two features of measurements of the first kind which Deutsch appeals to in his argument for Condition C1 on the interpretation basis are:

F1. If the object system is initially in an eigenstate of the observable being measured, then, on completion of the measurement, it should be in this same eigenstate and the apparatus subsystem should be in the eigenstate of the apparatus pointer observable with eigenvalue corresponding to the eigenvalue of the object system state. This is the defining property of measurements of the first kind. Interactions between the object and apparatus subsystems with this property can be described in the formalism of quantum mechanics by an appropriate Hamiltonian (von Neumann, 1955, Chapter VI, Section 3; Deutsch, 1985, p. 12) and therefore interactions of this type are allowed by the theory.

F2. The dependence of the apparatus observable on the object observable should cease on completion of the measurement. The measurement is set up to correlate eigenstates of the apparatus observable with eigenstates of the object observable; once this correlation has been achieved the interaction between the two subsystems should terminate. This signals the completion of the measurement as far as the MWI is concerned; the relevant information has been transmitted to the apparatus subsystem.

Deutsch's argument is then: suppose that initially the object system is in an eigenstate of the object observable, say $|a_1^i\rangle$; then it follows from F1 that on completion of the measurement, the object system is in the eigenstate $|a_1^i\rangle$ of the object observable, and the apparatus system is in the corresponding eigenstate of the apparatus pointer observable. Let this be written $|a_2^i\rangle$. Therefore the state of the object–apparatus system is $|a_1^i\rangle|a_2^i\rangle$, a factorizable, nonentangled state for the two subsystems. According to F2, the dependence of the apparatus observable on the object observable ceases after this time; therefore the system should remain in a factorizable state for the object and apparatus subsystems. The necessary and sufficient condition for this to occur, which is given by Deutsch in Section 3 and referred to in his heuristic argument, is that the Hamiltonian $\hat{H}$ for the joint system on completion of the measurement is such that $[\hat{H}-(\hat{H}_1+\hat{H}_2)]|a_1^i\rangle|a_2^i\rangle=0$, for some $\hat{H}_1$ and $\hat{H}_2$, Hamiltonians confined to the object and apparatus subsystems, respectively. However, in his heuristic argument he states the condition as: the Hamiltonian for the joint system on completion of the measurement is such that $|a_1^i\rangle|a_2^i\rangle$ is an eigenstate of $\hat{H}-(\hat{H}_1+\hat{H}_2)$ for some $\hat{H}_1$ and $\hat{H}_2$, Hamiltonians confined to the object and apparatus subsystems, respectively. This is perhaps a little confusing, but it is not incorrect, since the two conditions are in fact equivalent, as can be easily shown.

The same line of reasoning may be followed through for each object observable eigenstate $|a_1^i\rangle$ to show that the Hamiltonian for the joint system must be such that the following condition is satisfied: each $|a_1^i\rangle|a_2^i\rangle$ is an eigenstate of $\hat{H}-(\hat{H}_1+\hat{H}_2)$ for some $\hat{H}_1$ and $\hat{H}_2$, Hamiltonians confined to the object and apparatus subsystems, respectively. This condition is equivalent to the condition: for each $|a_1^i\rangle|a_2^i\rangle$, there exist some $\hat{H}_1$ and $\hat{H}_2$ such that $[\hat{H}-(\hat{H}_1+\hat{H}_2)]|a_1^i\rangle|a_2^i\rangle=0$. However, from these conditions it does not follow that each basis state $|a_1^i\rangle|a_2^j\rangle$ is an eigenstate of $\hat{H}-(\hat{H}_1+\hat{H}_2)$ for some $\hat{H}_1$ and $\hat{H}_2$, nor equivalently that for each basis state $|a_1^i\rangle|a_2^j\rangle$ there exist some $\hat{H}_1$ and $\hat{H}_2$ such that $[\hat{H}-(\hat{H}_1+\hat{H}_2)]|a_1^i\rangle|a_2^j\rangle=0$. In order to establish this condition, Deutsch appeals to a "more realistic model" of measurement, where imperfections in the measurement allow for each of the basis states, $|a_1^i\rangle|a_2^j\rangle$ to appear as final states, not just the accurate ones, $|a_1^i\rangle|a_2^i\rangle$. Feature F2, that on completion of the measurement the apparatus observable should cease to depend on the object observable, is still required of such measurements and so, following through the same line of argument as for the accurate states, the following condition is obtained: each basis state $|a_1^i\rangle|a_2^j\rangle$ is an eigenstate of $\hat{H}-(\hat{H}_1+\hat{H}_2)$ for some $\hat{H}_1$ and $\hat{H}_2$. This is equivalent to the condition: for each of the basis states $|a_1^i\rangle|a_2^j\rangle$, there exists some $\hat{H}_1$ and $\hat{H}_2$ such that $[\hat{H}-(\hat{H}_1+\hat{H}_2)]|a_1^i\rangle|a_2^j\rangle=0$.

Deutsch concludes that there must exist a choice of $\hat{H}_1$ and $\hat{H}_2$ such that $\hat{H}-(\hat{H}_1+\hat{H}_2)$ is diagonal in the interpretation basis $\{|a_1^i\rangle|a_2^j\rangle\}$, i.e., there must exist a choice of $\hat{H}_1$ and $\hat{H}_2$ such that

$$[\hat{H}-(\hat{H}_1+\hat{H}_2)]|a_1^i\rangle|a_2^j\rangle=\alpha^{ij}|a_1^i\rangle|a_2^j\rangle$$

for each state $|a_1^i\rangle|a_2^j\rangle$ in the interpretation basis. However, this condition does not follow from the condition argued for in the preceding paragraph: In general, from the condition, *for each i, j, there exists an x and y such that*..., it does not follow that, *there exists an x and y such that for each i, j*, ... We would have to prove that in this particular case, in the theory of quantum mechanics, the inference is valid. We conjecture that no such proof exists; the one does not follow from the other. Therefore, Deutsch's heuristic argument is inadequate as it stands to establish Condition C1 as a condition on the interpretation basis (even in the particular case considered on completion of a perfect nondisturbing measurement) and we conjecture that the gaps cannot be filled to render an adequate argument.

Condition C1 is a condition on both the product structure and the basis in that structure, but it is not sufficient to determine these. Deutsch claims that essentially it fixes the product structure. In arguing for a further condition, which with the first should determine the interpretation basis, Deutsch supposes that the product structure dividing a system into two subsystems is already given, and uses an argument based on external measurements to obtain a condition determining a basis for this product structure: C2. The interpretation basis at time t is the basis of eigenstates of $\hat{\rho}_1(t)$ $(=\mathrm{Tr}_{2,t}\,\hat{\rho})$ and of $\hat{\rho}_2(t)$ $(=\mathrm{Tr}_{1,t}\,\hat{\rho})$, where $\hat{\rho}=|\psi\rangle\langle\psi|$, $|\psi\rangle$ being the state of the joint system. A defender of a physical theory which takes quantum mechanics and combines it with some projection postulate interpreted as representing an objective collapse of the state function (a "collapse theory") could use an argument of the form given by Deutsch to justify taking this condition as a formal condition on the eigenstates into which the state of a system collapses on measurement.

Thus, suppose two systems which have interacted in the past so that they are in some entangled state $|\psi\rangle$ are now spatially separated. Assume that a measurement on one effects a discontinuous change of the state vector for the joint system into a mixture described by some density operator,

$$\hat{\rho}_m = \sum_{ij} |\langle\psi|a_1^i a_2^j\rangle|^2 |a_1^i a_2^j\rangle\langle a_1^i a_2^j|$$

where $\{|a_1^i\rangle\}$ is some complete set of orthonormal states in the Hilbert space of subsystem 1 and $\{|a_2^j\rangle\}$ is some complete set of orthonormal states in the Hilbert space of subsystem 2. We want to determine the conditions which must be satisfied by this mixture if there is to be no superluminal signaling, or, in general, no signaling without some interaction described by the dynamical equations of quantum mechanics.

To remain consistent with the prohibition of the propagation of information at superluminal velocities or by nondynamical means, measurements performed at time t on one of the subsystems must not affect the probabilities of the results of measurements performed at the "same" time on the other subsystem. If the state vector is not collapsed before measurements are made on subsystem 1, then the statistics for these measurement results, whichever $\hat{\xi}_2(t)$ observable is to be measured on subsystem 2, can be calculated from the density operator $\hat{\rho}(t) = |\psi\rangle\langle\psi|$, where $|\psi\rangle$ is the uncollapsed state vector at time t, by tracing over the subsystem 2 states. On the other hand, if the subsystem 2 measurement has been made and the state vector has collapsed into a mixture, then the statistics for the subsystem 1 measurement results can be obtained from the density operator $\hat{\rho}_m(t)$ by tracing over the subsystem 2 states. If the statistics obtained from $\hat{\rho}_m$ were different from the statistics obtained from $\hat{\rho}$, it would be possible in principle to determine at time t from measurements on subsystem 1 whether or not a measurement had been made at a spacelike separation from subsystem 1, or, in general, on a subsystem no longer dynamically interacting with subsystem 1. Therefore, if there is to be no superluminal signaling or no nondynamically carried transfer of information, $\hat{\rho}_{m1}(t)$ $(=\mathrm{Tr}_{2,t}\,\hat{\rho}_m)$ must be equal to $\hat{\rho}_1(t)$ $(=\mathrm{Tr}_{2,t}\,\hat{\rho})$. Therefore, since $\hat{\rho}_{m1}(t)$ is diagonal in the interpretation basis, $\hat{\rho}_1(t)$ must be diagonal in the interpretation basis. A similar argument applies to $\hat{\rho}_2(t)$ $(=\mathrm{Tr}_{2,t}\,\hat{\rho})$. Therefore, the states which make up the mixture into which the state of the system collapses must be the product states of the eigenstates of $\hat{\rho}_1$ with the eigenstates of $\hat{\rho}_2$. Call this Condition C2′.

It might be suggested that this argument for Condition C2′ is redundant given Furry's (1936) work on the distinction between a mixture and the pure state for a system of two correlated subsystems, because from his results it can be shown that it is already guaranteed that no violation of the "no superluminal signaling principle" will occur. But from Furry's work it can only be shown that no violation occurs, supposing the *particular* mixture he gives forms as a result of a measurement on one of the subsystems. This mixture happens to satisfy Condition C2′. It cannot be concluded from this that *any* mixture assumed on collapse *must* satisfy Condition C2′ if no violation is to occur. It is this conclusion which is required to establish condition C2′ and which is the conclusion of the argument given above.

However, an argument of the same form to establish that C2 is a necessary condition on the states in the interpretation basis does not follow through in the context of the MWI. Although in the MWI Deutsch introduces a density operator $\hat{\rho}_\varepsilon$ which, on completion of a measurement, looks identical to $\hat{\rho}_m$ (the density operator in the collapse theory), $\hat{\rho}_\varepsilon$ is given a very different physical significance than $\hat{\rho}_m$ and other density operators in quantum mechanics. This renders invalid a crucial step in the argument when it is translated from the collapse version to a MWI version.

According to Deutsch's version of the MWI, the world consists of a continuously infinite measured set of universes (that is, a set together with a measure on that set) such that at each instant t, in a proportion $|\langle\psi|\alpha, t\rangle|^2$ of the set of universes, the value of an observable $\hat{O}$ diagonal in the interpretation basis $\{|\alpha, t\rangle\}$ is $\langle\alpha, t|\hat{O}|\alpha, t\rangle$. Deutsch claims that this description may be summarized as follows: If the interpretation basis at time t is $\{|\alpha, t\rangle\}$, then the set of all universes at time t is an ensemble which may be described by the density operator

$$\hat{\rho}_\varepsilon(t) = \sum_\alpha |\langle\psi|\alpha, t\rangle|^2 |\alpha, t\rangle\langle\alpha, t|$$

(Deutsch, 1985, pp. 20, 21). But if this is intended only as a summary of the previous description, then $\hat{\rho}_\varepsilon$ cannot be given the same physical significance as is usually given to quantum mechanical density operators. Normally, the statistics calculated from the quantum mechanical density operator for a system for *any* observable of the system is given a physical significance, in that it is interpreted as the *predicted* statistics for the results of a measurement of that observable. $\hat{\rho}_\varepsilon$, however, is only supposed to summarize the statistics of *certain* observables of the system, those that are diagonal in the interpretation basis, and the statistics given is interpreted as describing the distribution of *possessed* values. $\hat{\rho}_\varepsilon$ is not supposed to tell us anything about observables not diagonal in the interpretation basis. If the projection operator for the eigenvalue q_i of the observable $\hat{Q}$ is $\hat{P}(q_i)$, then, if $\hat{Q}$ is diagonal in the interpretation basis at time t, $\mathrm{Tr}[\hat{\rho}_\varepsilon \cdot \hat{P}(q_i)]$ is the proportion of universes in which the observable $\hat{Q}$ has the value q_i; if $\hat{Q}$ is not diagonal in the interpretation basis, $\mathrm{Tr}[\hat{\rho}_\varepsilon \cdot \hat{P}(q_i)]$ has no physical significance; it does not give the proportion of universes in which a certain value is possessed by the observable Q, nor the proportion of universes in which a certain value will be obtained on measurements of Q.

Now we can see that the translation of the argument for Condition C2′ in the context of a collapse theory to the context of the MWI to generate an argument for Condition C2 is invalid because the following step in the MWI version is invalid: if the subsystem 2 measurement has been made and the system is described at time t by the density operator $\hat{\rho}_\varepsilon(t)$, then the statistics for the subsystem 1 measurement results can be obtained by tracing over the states for all the subsystems except subsystem 1. This is invalid because $\hat{\rho}_\varepsilon(t)$ is only to be used to give the statistics of observables diagonal in the interpretation basis, not just any subsystem 1 observable. Therefore, Deutsch's argument for Condition C2 does not carry through in the context of the MWI. Of course, $\hat{\rho}_\varepsilon$ should give the same statistics as $\hat{\rho}$ for observables which *are* diagonal in the interpretation basis, but this is already guaranteed by the way in which $\hat{\rho}_\varepsilon$ is constructed: Let the interpretation basis at time t be $\{|\alpha, t\rangle\}$, and let the expansion of $|\psi, t\rangle$ in the interpretation basis be $|\psi, t\rangle = \sum_\alpha c_\alpha |\alpha, t\rangle$; then

$$\hat{\rho}(t) = \sum_{\alpha,\beta} c_\alpha c_\beta^* |\alpha, t\rangle\langle\beta, t|$$

$$\hat{\rho}_\varepsilon(t) = \sum_\alpha |c_\alpha|^2 |\alpha, t\rangle\langle\alpha, t|$$

Let $\hat{Q}$ be an observable which is diagonal in the interpretation basis, and let its eigenvalue for eigenstate $|\alpha\rangle$ be q_α. Then, using $\hat{\rho}$, the probability that on measurement $Q \stackrel{?}{=} q_\alpha$ is $\mathrm{Tr}[|\alpha\rangle\langle\alpha| \cdot \hat{\rho}]$, which equals $|c_\alpha|^2$. In the MWI, using $\hat{\rho}_\varepsilon$, the proportion of universes in which Q has the value q_α is $\mathrm{Tr}[|\alpha\rangle\langle\alpha| \cdot \hat{\rho}_\varepsilon]$, which also equals $|c_\alpha|^2$.

Any argument that tries to establish that $\hat{\rho}_i$ should equal $\hat{\rho}_{\varepsilon i}$, i.e., that the statistics calculated from $\hat{\rho}_i$ for any observable Q should equal those calculated from $\hat{\rho}_{\varepsilon i}$ for Q, $i = 1, 2$, is going to fall prey to the objection that $\hat{\rho}_\varepsilon$ should only be used to give the statistics of observables diagonal in the interpretation basis, unless some physical reasons can be given for interpreting $\hat{\rho}_\varepsilon$ differently. Without such reasons, in order to defend Condition C2, that the interpretation basis at time t is the basis of eigenstates of $\hat{\rho}_1(t)$ and $\hat{\rho}_2(t)$, in the context of the MWI, we must find an argument which does not have to establish this identity. One way one might defend Condition C2 is to point out that in the case of a perfect nondisturbing measurement the basis we want for the interpretation basis satisfies Condition C2. But this does not give us a physical explanation as to why the condition should hold in this case, let alone in general.

To conclude this section, Deutsch's heuristic argument is inadequate to set up either Condition C1 or C2 for use in his algorithm. If Deutsch still wants to present a heuristic argument for these conditions, it must take some other form. A line of argument, which Deutsch himself suggests

(Deutsch, 1985, p. 29), and perhaps should be considered further, uses the idea that these conditions are the requirements that the different world branches cannot communicate with one another under most circumstances. But we leave a consideration of this proposal to another time.

2.2. The Existence and Uniqueness of a Suitable Solution to Deutsch's Algorithm

1. In generalizing his method to the case of a system the dimensionality of whose Hilbert space has arbitrarily (but finitely) many factors, Deutsch assumes that the interpretation basis product structure should divide the system into subsystems with prime-dimensional Hilbert spaces, and that his algorithm finally determines such a product structure through repeated use of Condition C1. The question therefore arises as to whether these two assumptions are justified. We see no reason why the subsystems of the interpretation basis should always have to have Hilbert spaces of prime dimension, though they might. But supposing, as Deutsch does, that they do, we want to check that a final solution to Deutsch's algorithm always exists. A final solution will always exist if, for an arbitrary system with a Hilbert space of dimension mn, for any Hamiltonian $\hat{H}$ and state $|\psi\rangle$ for the system, there is always some product structure dividing the system into two subsystems 1 and 2, one with a Hilbert space of dimension m, the other with a Hilbert space of dimension n, which satisfies Condition C1. In the final stage of applying the algorithm, m and n will both be prime. In the preceding stages, only one of them will be prime. Condition C1 states that there exist $\hat{H}_1$ and $\hat{H}_2$, Hermitian operators in the Hilbert spaces of subsystems 1 and 2, respectively, such that $\hat{H}-\hat{H}_1-\hat{H}_2$ is diagonal in the interpretation basis, which, according to Condition C2, is the basis in which the subsystem density operators $\hat{\rho}_1$ and $\hat{\rho}_2$ are diagonal. In his paper Deutsch (1985) gives the following "crude but encouraging argument" for the existence of a (unique) product structure satisfying Condition C1.

Suppose Conditions C1 and C2 hold for some product structure dividing the system into two subsystems 1 and 2; then, for some $\hat{H}_1$ and $\hat{H}_2$, $\hat{H}-\hat{H}_1-\hat{H}_2$ is diagonal in the basis in which $\hat{\rho}_1$ and $\hat{\rho}_2$ are diagonal. From this it follows that, for some $\hat{H}_1$ and $\hat{H}_2$,

$$[\hat{H}-\hat{H}_1-\hat{H}_2, \hat{\rho}_1\otimes\hat{\rho}_2]=0 \tag{E1}$$

By taking the partial traces of this equation for subsystems 1 and 2, we obtain the following equations:

$$\mathrm{Tr}_{1,t}[\hat{H}, \hat{\rho}_1\otimes\hat{\rho}_2]=[\hat{H}_2, \hat{\rho}_2]$$

$$\mathrm{Tr}_{2,t}[\hat{H}, \hat{\rho}_1\otimes\hat{\rho}_2]=[\hat{H}_1, \hat{\rho}_1]$$

If these are substituted back into the original equation, we obtain the following equation:

$$[\hat{H}, \hat{\rho}_1 \otimes \hat{\rho}_2] - \hat{\rho}_1 \otimes \mathrm{Tr}_{1,t}[\hat{H}, \hat{\rho}_1 \otimes \hat{\rho}_2] - \mathrm{Tr}_{2,t}[\hat{H}, \hat{\rho}_1 \otimes \hat{\rho}_2] \otimes \hat{\rho}_2 = 0 \quad \text{(E2)}$$

Since this equation has the same number of independent real equations as independent components in the product structure, it should have a unique solution. The hope is that therefore there always exists a (unique) product structure satisfying Condition C1.

However, $\hat{H}$ and $|\psi\rangle$ may be such that there is a product structure and $\hat{H}_1$ and $\hat{H}_2$ satisfying equation (E1) and therefore equation (E2), but that in this product structure, $\hat{H} - \hat{H}_1 - \hat{H}_2$ is not diagonal in the interpretation basis in which $\hat{\rho}_1$ and $\hat{\rho}_2$ are diagonal; $\hat{H} - \hat{H}_1 - \hat{H}_2$ may have some nonzero off-diagonal elements and still satisfy equation (E1) as long as $\hat{\rho}_1 \otimes \hat{\rho}_2$ has some degeneracies. These nonzero elements need not be such that it is possible to find some other $\hat{H}'_1$ and $\hat{H}'_2$ such that $\hat{H} - \hat{H}'_1 - \hat{H}'_2$ is diagonal in the basis in which $\hat{\rho}_1$ and $\hat{\rho}_2$ are diagonal. Therefore, although a product structure satisfying Condition C1 will satisfy equations (E1) and (E2), a product structure satisfying equations (E1) and (E2) does not necessarily satisfy Condition C1. Therefore, Deutsch's argument that a unique solution exists for equation (E2) does not necessarily show that a solution always exists for Condition C1, and so he does not have an argument to show that a final solution always exists to his algorithm. Therefore, there may be situations in which there is no solution to his algorithm and no interpretation basis will be determined. We consider next whether, if a solution does exist, it is guaranteed to be unique.

2. In his paper, Deutsch (1985) admits that degeneracies in $\hat{\rho}_1(t)$, $\hat{\rho}_2(t)$, or $\hat{H}(t) - \hat{H}_1(t) - \hat{H}_2(t)$ will occur in some cases and spoil the uniqueness of the solution to his algorithm; his algorithm will pick out more than one interpretation basis. For each interpretation basis there will be on offer an apparently different description as to how the world is in such cases. However, Deutsch (1985, p. 28) claims that: "Such degeneracies occur precisely when there is nothing in the world to distinguish between the different interpretations arising from the equivalence class of interpretation bases generated by the algorithm. The interpretations differ in form only."

In this section we consider various situations in which $\hat{\rho}_1(t)$ and $\hat{\rho}_2(t)$ are degenerate and discuss in each case whether Deutsch's claim is acceptable. We show that the degeneracies occur even for situations in which it appears that the different interpretations on offer should not be construed as differing in form only, and therefore that Deutsch's algorithm is inadequate to pick out the required interpretation basis in these situations.

The example of degeneracy which Deutsch discusses in his article is the case of two spin-$\frac{1}{2}$ systems 1 and 2 in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2 + |\downarrow\rangle_1|\uparrow\rangle_2)$$

Assuming that the correct product structure divides the system into the two spin-$\frac{1}{2}$ subsystems 1 and 2, $\hat{\rho}_1 = \frac{1}{2}(|\uparrow\rangle_1\langle\uparrow|_1 + |\downarrow\rangle_1\langle\downarrow|_1)$, and is therefore degenerate; we can take as orthonormal eigenstates $|\uparrow\rangle_1$ and $|\downarrow\rangle_1$, or

$$|N\rangle_1 = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 + |\downarrow\rangle_1), \qquad |S\rangle_1 = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 - |\downarrow\rangle_1)$$

or

$$|+\theta\rangle_1 = \cos\theta\,|\uparrow\rangle_1 + \sin\theta\,|\downarrow\rangle_1, \qquad |-\theta\rangle_1 = \sin\theta\,|\uparrow\rangle_1 - \cos\theta\,|\downarrow\rangle_1$$

or ...... Also, $\hat{\rho}_2 = \frac{1}{2}(|\uparrow\rangle_2\langle\uparrow|_2 + |\downarrow\rangle_2\langle\downarrow|_2)$ and is therefore degenerate; we can take as eigenstates $|\uparrow\rangle_2$ and $|\downarrow\rangle_2$, or $|N\rangle_2$ and $|S\rangle_2$, or $|+\theta\rangle_2$ and $|-\theta\rangle_2$, or .... Therefore, we can take as interpretation basis the product states of $|\uparrow\rangle_1, |\downarrow\rangle_1$ and $|\uparrow\rangle_2, |\downarrow\rangle_2$, or $|N\rangle_1, |S\rangle_1$ and $|N\rangle_2, |S\rangle_2$, or $|\uparrow\rangle_1, |\downarrow\rangle_1$ and $|N\rangle_2, |S\rangle_2$, or $|\uparrow\rangle_1, |\downarrow\rangle_1$ and $|+\theta\rangle_2, |-\theta\rangle_2$, etc.

Depending on which of these possible interpretation bases we choose, we obtain a "different" description of the world. For example, choosing the basis $\{|\uparrow\rangle_1|\uparrow\rangle_2, |\uparrow\rangle_1|\downarrow\rangle_2, |\downarrow\rangle_1|\uparrow\rangle_2, |\downarrow\rangle_1|\downarrow\rangle_2\}$, the interpretation is that the world is divided into two branches of equal size: in one, subsystem 1 has spin up, subsystem 2 spin down; in the other branch, subsystem 1 has spin down and subsystem 2 has spin up. Choosing the basis $\{|N\rangle_1|N\rangle_2, |N\rangle_1|S\rangle_2, |S\rangle_1|N\rangle_2, |S\rangle_1|S\rangle_2\}$, the world is divided into two branches of equal size: in one, both subsystems have spins in the direction north; in the other branch, both subsystems have spins in the direction south. Choosing the basis $\{|\uparrow\rangle_1|N\rangle_2, |\uparrow\rangle_1|S\rangle_2, |\downarrow\rangle_1|N\rangle_2, |\downarrow\rangle_1|S\rangle_2\}$, the expansion of $|\psi\rangle$ is

$$|\psi\rangle = \tfrac{1}{2}(|\uparrow\rangle_1|N\rangle_2 + |\uparrow\rangle_1|S\rangle_2 + |\downarrow\rangle_1|N\rangle_2 - |\downarrow\rangle_1|S\rangle_2)$$

The interpretation is therefore that the world is divided into four branches: in one, subsystem 1 has spin up and subsystem 2 spin north; in a second branch, subsystem 1 has spin up and subsystem 2 has spin south; in a third branch, subsystem 1 has spin down and subsystem 2 has spin north; in the fourth branch, subsystem 1 has spin down and subsystem 2 has spin south. Choosing the basis $\{|\uparrow\rangle_1|+\theta\rangle_2, |\uparrow\rangle_1|-\theta\rangle_2, |\downarrow\rangle_1|+\theta\rangle_2, |\downarrow\rangle_1|-\theta\rangle_2\}$, the expansion of $|\psi\rangle$ is

$$|\psi\rangle = \frac{1}{\sqrt{2}}(\sin\theta|\uparrow\rangle_1|+\theta\rangle_2 - \cos\theta|\uparrow\rangle_1|-\theta\rangle_2$$
$$+ \cos\theta\,|\downarrow\rangle_1|+\theta\rangle_2 + \sin\theta\,|\downarrow\rangle_1|-\theta\rangle_2)$$

Therefore, except when $\theta = n\pi/2$, the interpretation is that the world is divided into four branches, which are not all of equal size unless $\theta = \pi/4 + n\pi/2$: in one, subsystem 1 has spin up and subsystem 2 has spin in the plus direction defined by θ; in another, subsystem 1 has spin up and subsystem 2 has spin in the minus direction defined by θ, etc.

If we consider the singlet state,

$$|\psi_s\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2)$$

instead of the triplet state $|\psi\rangle$, the same subsystem density operators are obtained. Therefore, the same degeneracies occur and the same set of interpretation bases is given by the algorithm. Again some of the bases generate two-branch interpretations, others four-branch interpretations. But in this case, due to the spherical symmetry of the singlet state, all the two-branch interpretations have a similar form—the spins are in opposite directions in each of the branches.

These two examples invalidate Deutsch's claim (Deutsch, 1985, p. 30) that Condition C2 on the interpretation basis implies that in the interpretation basis, the matrix representing the state $|\psi\rangle$ has no more than one nonzero element per row or column, making it possible to relabel the interpretation basis states so that the matrix is diagonal and $|\psi\rangle$ takes the Schmidt normal form in this basis, i.e., is of the form $|\psi\rangle = \sum_i c_i|a_1^i|a_2^i\rangle$, where $\{|a_1^i\rangle\}$ is some orthonormal set of eigenstates in H_1, the Hilbert space for subsystem 1, and $\{|a_2^i\rangle\}$ is some orthonormal set of eigenstates in H_2, the Hilbert space for subsystem 2. The claim is valid only in the case that $\hat{\rho}_1$ and $\hat{\rho}_2$ are both nondegenerate. Condition C2 then determines a unique interpretation basis, and if we expand $|\psi\rangle$ in terms of this basis, we obtain $|\psi\rangle$ in the Schmidt normal form for the subsystems 1 and 2. However, if one of the $\hat{\rho}_i$ is degenerate, then it does not have a unique set of eigenstates. Deutsch's algorithm gives as interpretation basis the product states of *any* of the complete sets of eigenstates of the degenerate density operator with the set of eigenstates of the other density operator. Therefore, a number of interpretation bases are given by the algorithm. If $|\psi\rangle$ is expanded in each of these bases, only one will express $|\psi\rangle$ in Schmidt normal form; the other bases will expand $|\psi\rangle$ in some form $|\psi\rangle = \sum_{ij} c_{ij}|b_1^i\rangle|b_2^j\rangle$, where $\{|b_1^i\rangle\}$ is some orthonormal set of eigenstates in the Hilbert space for subsystem 1 and $\{|b_2^j\rangle\}$ is some orthonormal set of eigenstates in the Hilbert space for subsystem 2 such that the matrix constructed from the coefficients c_{ij} has at least one row or column in which there is more than one nonzero coefficient. If both $\hat{\rho}_1$ and $\hat{\rho}_2$ are degenerate, then neither of them has a unique set of eigenstates. Deutsch's algorithm gives as interpretation basis the product states of *any* complete set of eigenstates of $\hat{\rho}_1$ with *any* complete set of eigenstates for $\hat{\rho}_2$. Therefore, a number of interpretation bases are given by the algorithm. If $|\psi\rangle$ is expanded in each of these bases, some of them will express $|\psi\rangle$ in Schmidt normal form, others will not, and for these interpretation bases the matrix constructed from the coefficients will have at least one row or column in which there is more than one nonzero coefficient. Notice that if any of the nonzero coefficients in a Schmidt normal form expansion of $|\psi\rangle$ for a certain product structure are of equal modulus, the Schmidt normal form is not unique for that product structure. The subsystem density operators will be degenerate and Condition C2 will not determine a unique basis for each of them.

Concerning the nonuniqueness of the interpretation basis for the triplet state, Deutsch (1985, p. 28) writes:

> The possibility of more than one interpretation is not consistent with the principle of realism which we have been implementing everywhere. But of course the terms "up," "down," "North," and "South" have no invariant meaning in the simple world of (57). They would acquire meaning only if the degeneracy were broken by the dynamical evolution.

The idea is that, since in order to define the physical directions some physical system must be taken as a reference system, in the simple world of $|\psi\rangle$, where there is not another system to act as a reference system for the two spin-$\frac{1}{2}$ systems relative to which a direction to call "up," a different direction to call "north," etc., can be defined, it does not matter that our mathematical model for this situation offers us a number of alternative representations; there is no physical difference between these representations in this case.

However, as described in his paper, Deutsch's (1985) version of the MWI gives us a picture of the world as a set of universes differentiated at any time into a number of distinct subsets, i.e., branches. If we adopt this realist picture, then, presumably, the number of distinct branches at any time should be definite and if the quantum mechanical description is complete, then the state and the interpretation basis, determined by the state, Hilbert space, and Hamiltonian for the system, should determine this number. However, for state $|\psi\rangle$, some of the interpretation bases on offer distinguish two branches, some four. Therefore, if we are not to abandon the program altogether, either we maintain all the interpretation bases and modify our picture of the world or try somehow to reduce the set of interpretation bases on offer, at least to a set which generates the same number of branches.

Deutsch has suggested (private communication) a modification to the many universes picture of the world which maintains all the interpretation bases on offer. We will not involve ourselves here in either a description of this modification or a discussion of whether it constitutes a feasible picture for a realist to hold. For, in Section 2.3, we show that there are other situations in which degeneracies in the subsystem density operators occur and for which it can be argued that a unique basis should be picked out for the interpretation basis. We show that Deutsch's algorithm is inadequate to do this. Therefore, we conclude that any interpretation employing this algorithm will be inadequate.

2.3. A Major Inadequacy in Deutsch's Algorithm

Consider the following model of a simple measurement of the first kind: the $|0\rangle_1, |1\rangle_1$ states of subsystem 1, the "object" system, are to be "measured" by the $|0\rangle_2, |1\rangle_2$ states of subsystem 2, the "apparatus" system. Both systems have two-dimensional Hilbert spaces. The "measurement" interaction is such that $|i\rangle_1|0\rangle_2 \to |i\rangle_1|i\rangle_2$. So, if we start with the object subsystem in the superposition

$$|+\rangle_1 = \frac{1}{\sqrt{2}}(|0\rangle_1 + |1\rangle_1)$$

and the apparatus subsystem in the "receptive" state $|0\rangle_2$, then the state of the combined system at the end of the measurement will be

$$\frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2)$$

Assuming that the correct product structure divides the system into the object and apparatus subsystems, we obtain degenerate subsystem density operators, just like in the case of $|\psi\rangle$ and $|\psi_s\rangle$. Therefore, Deutsch's algorithm does not pick out a unique interpretation basis. Among the interpretation bases on offer are: the product states of $\{|0\rangle, |1\rangle_1\}$ and $\{|0\rangle_2, |1\rangle_2\}$, the product states of $\{|\pm\rangle_1\}$ and $\{|\pm\rangle_2\}$, the product states of $\{|0\rangle_1, |1\rangle_1\}$ and $\{|\pm\rangle_2\}$, etc., where $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$.

Given that the setup was supposed to describe a measurement of the first kind of the subsystem 1 observable $\hat{O} = |1\rangle_1\langle 1|_1$ by the subsystem 2 observable $\hat{P} = |1\rangle_2\langle 1|_2$, a suitable algorithm for defining the interpretation basis should pick out the basis of product states of the eigenstates of these two observables uniquely, in order for these observables to be given definite (multi-) values according to the interpretation. However, as has been shown above, Deutsch's algorithm does not pick out this basis uniquely. Indeed, some of the bases given by his algorithm pick out neither P nor O to have well-defined values. Some of the bases on offer (for example, the third listed above) give an expansion of the final object-apparatus state which has four elements with nonzero coefficients. For these bases the object and apparatus observables picked out to have well-defined values are not perfectly correlated in the final state. Each of the two values of the object observable occur with each of the two values of the apparatus observable. Therefore, according to these bases, *no* measurement of the first kind of *any* subsystem 1 observable by *any* subsystem 2 observable has occurred.

Now, either a measurement of the first kind is achieved for the system described in the simple model or it is not. If a measurement of the first kind is achieved, then, in order to maintain the claim that quantum mechanics interpreted using the interpretation basis determined by Deutsch's algorithm is a complete theory, we need to find a way to reduce the number of interpretation bases on offer for the system on completion of the measurement. If a measurement of the first kind is not achieved in the situation described in the simple model, then, assuming that measurements of the first kind are possible, even for the case when the initial state of the object system is a superposition of eigenstates of the operator corresponding to the observable being measured with some coefficients equal (let us call such measurements "equal coefficient measurements of the first kind"), we need to show how the simple model should be elaborated to break the degeneracy and pick out a suitable interpretation basis so that a model may be provided for such equal coefficient measurements. If neither of these alternative courses of action succeeds, Deutsch's algorithm is inadequate to determine the basis we require for equal coefficient measurements of the first kind, and is therefore inadequate to determine the interpretation basis.

Alternative 1

Confronted just with a system composed of two subsystems in a state whose Schmidt normal form for the two subsystems has some nonzero coefficients of equal modulus, Deutsch's algorithm does not pick out a unique basis. On offer are not only those bases which generate other expansions of the state vector in Schmidt normal form for the two subsystems, i.e., bases formed from the product states of eigenstates of pairs of observables which are perfectly correlated in the joint state, but also bases which expand the state vector in the product states of eigenstates of pairs of observables which are not perfectly correlated in the joint state. However, if such a state arises as a result of a measurement of the first kind, we want to pick out a particular basis for the interpretation basis—the basis of product states of the eigenstates of the object observable being measured and eigenstates of the apparatus pointer observable. Perhaps we need to look at the historical development of the state vector for the object-apparatus system to its final state in order to pick out the appropriate basis using Deutsch's algorithm; perhaps when the state has evolved from a factorizable state for the two subsystems, as in the case of a measurement, the interaction which entangles the states determines a preferred basis for the end of the measurement. Suppose we apply the algorithm at time t, where $t < \tau$, the time at which the measurement is complete; perhaps this will give a unique interpretation basis $\{|\alpha_t, t\rangle\}$. If we calculate this basis for t as $t \to \tau$, perhaps we will find it converging to the basis which we want to pick out for time τ, the end of the measurement. Let us investigate this idea for the simple model of a measurement of the first kind given earlier.

Now, in order to calculate the interpretation basis at an intermediate stage in the measurement, we need to find the Hamiltonian for the measurement interaction. This is not defined uniquely by the condition $|i\rangle_1|0\rangle_2 \to |i\rangle_1|i\rangle_2$ given earlier. We need to specify whether the interaction is constant during the time interval $t=0$ to $t=\tau$ and what happens to the states $|i\rangle_1|1\rangle_2$. Let us take the constant interaction, which is defined by $|i\rangle_1|j\rangle_2 \to |i\rangle_1|i \oplus j\rangle_2$, where $i=0,1, j=0,1$, and $\oplus \equiv$ addition modulus 2. Assuming that the self-interactions of the subsystems in the time interval $t=0$ to $t=\tau$ are negligible, it follows from the above that

$$\hat{H} = -\frac{\pi}{2\tau}|1\rangle_1\langle 1|_1(|0\rangle_2 - |1\rangle_2)(\langle 0|_2 - \langle 1|_2)$$

Then, if

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|0\rangle_1 + |1\rangle_1)|0\rangle_2$$

it follows that

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}[|0\rangle_1|0\rangle_2 + \tfrac{1}{2}(1+e^{(i\pi/\tau)t})|1\rangle_1|0\rangle_2 + \tfrac{1}{2}(1-e^{(i\pi/\tau)t})|1\rangle_1|1\rangle_2]$$

and

$$|\psi(\tau)\rangle = \frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2)$$

The question is whether applying the algorithm to $\hat{H}(t)$ and $|\psi(t)\rangle$ for $t<\tau$ will pick out a unique interpretation basis which converges on the basis $\{|0\rangle_1|0\rangle_2, |0\rangle_1|1\rangle_2, \ldots\}$ at the end of the measurement. It becomes apparent that the answer to this question is no, without doing the calculation, if we just express $\hat{H}(t)$, $|\psi(t)\rangle$, etc., in the basis $\{|+\rangle_1|+\rangle_2, |+\rangle_1|-\rangle_2, |-\rangle_1|+\rangle_2, |-\rangle_1|-\rangle_2\}$, where $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$. For then

$$\hat{H} = -\frac{\pi}{2\tau}(|+\rangle_1 - |-\rangle_1)(\langle +|_1 - \langle -|_1)|-\rangle_2\langle -|_2$$

$$|\psi(0)\rangle = |+\rangle_1 \frac{1}{\sqrt{2}}(|+\rangle_2 + |-\rangle_2)$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}[|+\rangle_1|+\rangle_2 + \tfrac{1}{2}(1+e^{(i\pi/\tau)t})|+\rangle_1|-\rangle_2 + \tfrac{1}{2}(1-e^{(i\pi/\tau)t})|-\rangle_1|-\rangle_2]$$

$$|\psi(\tau)\rangle = \frac{1}{\sqrt{2}}(|+\rangle_1|+\rangle_2 + |-\rangle_1|-\rangle_2)$$

In the (0–1) representation the process looks like a measurement of the $|0\rangle$, $|1\rangle$ states of subsystem 1 by the $|0\rangle$, $|1\rangle$ states of subsystem 2 and we would hope to be able to pick out the (0–1) basis at the end of the measurement. But in the ($\pm$) representation, the process looks like a measurement of the $|\pm\rangle$ states of subsystem 2 by the $|\pm\rangle$ states of subsystem 1, in which case we would want to pick out the ($\pm$) basis at the end of the measurement. Therefore, by looking at the historical development of the system to its final state, the most we will be able to do is reduce the bases on offer for the final state to the following two: the $\{|0\rangle_1, |1\rangle_1\} \otimes \{|0\rangle_2, |1\rangle_2\}$ basis and the $\{|\pm\rangle_1\} \otimes \{|\pm\rangle_2\}$ basis, both of which generate expansions of the final state in Schmidt normal form. Nothing in $\hat{H}$, $|\psi\rangle$, or H can make one of these representations more privileged than the other; they have corresponding features. If we introduce extra assumptions to pick out the (0–1) basis, we could equally well have introduced a corresponding set of assumptions to pick out the ($\pm$) basis. Therefore, Deutsch will not be able to pick out a unique interpretation basis for this simple measurement of the first kind using his algorithm without introducing arbitrary assumptions.

Deutsch's answer may be that in the situation described, there really is no physical difference between the two descriptions, no way to distinguish which of the two subsystems is to be called "1" and which is to be called "2" and no way to pick out the "0–1 direction" as opposed to the "$\pm$ direction." But then it needs to be shown how a model can ever be provided, using Deutsch's algorithm, for a measurement of the first kind, of a *specific* observable on an object in a state which is a superposition, with some coefficients equal, of eigenstates of this observable.

Alternative 2

In the following we explore various ways of elaborating the simple model in an attempt to find a model for equal coefficient measurements of the first kind, using Deutsch's algorithm. The idea is to see if by introducing a third subsystem the degeneracy can be broken. The presence of a third subsystem which has not and does not interact with the two subsystems 1 and 2 and so is not in an entangled state with these subsystems does nothing to remove the degeneracy; the density operator for subsystems 1 and 2 obtained by tracing over the state of the third subsystem is the same as before and so generates the same degeneracies. Therefore, we should investigate various ways of entangling a third subsystem, which it might be hoped will break the degeneracy.

(i) Suppose we introduce a third subsystem with a Hilbert space of dimension n and a basis in this Hilbert space $\{|i\rangle_3\}$, $i = 1, \ldots, n-1$. Suppose that this system is initially in the state $|0\rangle_3$ and interacts with subsystem 2 after 2 has interacted with subsystem 1, in such a way as to correlate its $|0\rangle_3$, $|1\rangle_3$ states with the $|0\rangle_2$, $|1\rangle_2$ states of subsystem 2, so that $|i\rangle_2|0\rangle_3 \to |i\rangle_2|i\rangle_3$. Given the initial state

$$\frac{1}{\sqrt{2}}(|0\rangle_1 + |1\rangle_1)|0\rangle_2|0\rangle_3$$

the final state of the three subsystems will then be

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2|0\rangle_3 + |1\rangle_1|1\rangle_2|1\rangle_3)$$

It follows that

$$\hat{\rho}_1 = \tfrac{1}{2}(|0\rangle_1\langle 0|_1 + |1\rangle_1\langle 1|_1)$$

$$\hat{\rho}_2 = \tfrac{1}{2}(|0\rangle_2\langle 0|_2 + |1\rangle_2\langle 1|_2)$$

$$\hat{\rho}_3 = \tfrac{1}{2}(|0\rangle_3\langle 0|_3 + |1\rangle_3\langle 1|_3)$$

Therefore, $\hat{\rho}_1$, $\hat{\rho}_2$ and $\hat{\rho}_3$ are all degenerate. Therefore, assuming that the correct product structure divides the system into the three subsystems 1, 2, and 3, if we apply Deutsch's algorithm for the case of three subsystems, whichever subsystem we deal with first, we obtain a degenerate density operator. Therefore, introducing a third subsystem which correlates its state with subsystem 2 produces a final three-subsystem state for which the problematic degeneracies still occur, and so does not give us a model for equal coefficient measurements of the first kind, using Deutsch's algorithm.

(ii) Suppose we introduce a third subsystem to measure a correlation observable on subsystems 1 and 2. The appropriate correlation observable to try, in the hope that the degeneracy will be broken, is

$$\hat{C} = |0\rangle_1|0\rangle_2\langle 0|_1\langle 0|_2 + |1\rangle_1|1\rangle_2\langle 1|_1\langle 1|_2$$

This has two pairs of degenerate eigenstates with eigenvalues 0 and 1. A complete set of orthonormal eigenstates for $\hat{C}$ is

$$\{|0\rangle_1|0\rangle_2,\ |0\rangle_1|1\rangle_2,\ |1\rangle_1|0\rangle_2,\ |1\rangle_1|1\rangle_2\}$$

The first and last of these states have eigenvalue 1, the other two have eigenvalue 0. Let subsystem 3 have at least three orthogonal states, $|R\rangle_3$, $|N\rangle_3$ and $|Y\rangle_3$. Let $|R\rangle_3$ be its receptive state prior to interaction with subsystems 1 and 2, $|N\rangle_3$ its state when C is found to equal 0, $|Y\rangle_3$ its state when C is found to equal 1. Let the interaction between subsystem 3 and subsystems 1 and 2 occur after subsystems 1 and 2 have interacted and be such that

$$|0\rangle_1|0\rangle_2|R\rangle_3 \to |0\rangle_1|0\rangle_2|Y\rangle_3$$

$$|0\rangle_1|1\rangle_2|R\rangle_3 \to |0\rangle_1|1\rangle_2|N\rangle_3$$

$$|1\rangle_1|0\rangle_2|R\rangle_3 \to |1\rangle_1|0\rangle_2|N\rangle_3$$

$$|1\rangle_1|1\rangle_2|R\rangle_3 \to |1\rangle_1|1\rangle_2|Y\rangle_3$$

Therefore, given the initial state we have been considering, the final state of the three subsystems will be

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2|Y\rangle_3 + |1\rangle_1|1\rangle_2|Y\rangle_3)$$

$$= \frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2\rangle|Y\rangle_3$$

Therefore, applying Deutsch's algorithm for three subsystems, assuming the correct product structure divides the system into the three subsystems 1, 2 and 3, taking subsystem 1, 2 or 3 first, we still obtain the problematic degeneracies for $\hat{\rho}_1$ and $\hat{\rho}_2$. Therefore, introducing a third subsystem to measure the correlation observable C on subsystems 1 and 2 generates a final state for which the problematic degeneracies still occur, and therefore does not seem to provide us with a model for equal coefficient measurements of the first kind.

Therefore, introducing a third subsystem in a manner which it might be hoped would pick out the appropriate basis for there to be an equal coefficient measurement of the first kind does not remove the degeneracy in the subsystem density operators for the final state. It can be seen that introducing further subsystems will be of no greater help in this. It seems that any elaboration on our simple model will not remove the degeneracies in the final state. Therefore, it seems that no model can be provided for equal coefficient measurements of the first kind, using Deutsch's algorithm.

3. CONCLUSION

In Section 2.1 it has been shown that Deutsch's heuristic argument is inadequate to establish the conditions appearing in his algorithm. In Section 2.2 it has been argued that a solution to Deutsch's algorithm may not always exist, and it has been shown that in some cases, although a solution exists, it is not unique. In Section 2.3 it has been argued that no model can be provided, using Deutsch's algorithm, for certain types of measurement and that, therefore, any interpretation using his algorithm will be inadequate to account for the achievement of such measurements. It cannot be concluded that there is no algorithm which, when employed with the MWI, can provide a model for such measurements, but the onus is on the defender of the MWI (or other interpretations which require an algorithm to determine a preferred basis) to come up with an adequate algorithm and an argument for it. Without a viable algorithm, we have to resort to *a priori* metaphysics to interpret any state function using these interpretations, and they lose one of their important selling points.

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